Five Year Integrated M.Sc. Examination, 2023-2024

Semester - V

Course: PH-3-5-1

Time: Four Hours | Classical Mechanics | Full Marks: 80

Questions are of values as indicated in the margin

Answer question number **one** and any **four** from the rest

1. Answer any **four** questions:

$4 \times 5 = 20$

(a) Derive Lagrange's equation of motion from variational principle.

(b) Find the degrees of freedom of a system of two point masses joined by a massless rigid rod in a 3-dimensional space. What will happen to the *d.o.f.* if we add another point mass with these two point masses by two massless rigid rods so as to form a triangular structure?

(c) Show that the total angular momentum of a system of particles about any arbitrary point is the sum of angular momentum due to a single particle of total mass of the system situated at the centre of mass and the angular momentum of the particles about the centre of mass.

(d) Define holonomic and nonholonomic constraints with examples.

(e) Let A and B are two constants of motion. Show that Poisson's bracket of A and B i.e. $\{A, B\}$, is another constant of motion.

(f) Show that the moment of inertia tensor is a symmetric tensor.

2. (a) A particle of mass m is moving under a force field $\vec{F} = -k\frac{\hat{x}}{x^3}$, where k is a positive constant. Construct the Lagrangian and obtain the Lagrange's equation of motion for the particle.

(b) Define cyclic coordinate. Show that the generalised momentum which is canonically conjugate to a cyclic coordinate, is a constant of motion.

(c) The Lagrangian of a particle of mass m is $L = \frac{1}{2}\left(m\dot{x}^2 - bx^2\right)e^{at}$, where a and b are positive constants. Determine the Hamiltonian. Is it a constant of motion?

5+5+5=15

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3. (a) Let S' be a reference frame which is rotating with respect to a fixed frame S with an angular velocity $\vec{\omega}$. Prove that for an arbitrary vector $\vec{A}$,

$$\frac{d\vec{A}}{dt} = \frac{d'\vec{A}}{dt} + \vec{\omega} \times \vec{A},$$

where $\frac{d\vec{A}}{dt}$ and $\frac{d'\vec{A}}{dt}$ refer to time derivatives with respect to S and S' frames, respectively.

   (b) Two reference frames, one is fixed and other one is rotating, have common origin. Obtain the equation of motion of a particle of mass m with respect to the rotating frame. Discuss about the different fictitious forces arise in the rotating frame.

   (c) Define Euler's angles with appropriate diagram. Why do you need three Euler's angles to describe the motion of a rigid body?

5+5+5=15

4. (a) Show that the Lagrange's equations of motion remain form invariant under the transformation :

$$L \to L' = L + \frac{dF(q,t)}{dt},$$

where $F(q,t)$ is a function of generalised coordinates (q_i) and time t.

   (b) Starting from Lorentz force law, show that the Lagrangian for a charged particle moving with velocity $\vec{v}$ in presence of electromagnetic field is given by

$$L = \frac{1}{2}m|\vec{v}|^2 - e\phi + e(\vec{v}.\vec{A}),$$

where ϕ and $\vec{A}$ are scalar and vector potential respectively.

   (c) Calculate the generalised momentum and construct the Hamiltonian of the charged particle described by the above-mentioned Lagrangian.

5+5+5=15

5. (a) Derive Euler's equations of motion for a torque free motion of a rigid body. Hence, show that for a symmetrical top, the angular velocity vector performs uniform precession about the symmetry axis of the body.

   (b) Find out the conditions on α and β for which following transformation represents a canonical transformation.

$$Q = q^\alpha \cos \beta p, \quad P = q^\alpha \sin \beta p.$$

(5+5)+5=15

6. (a) Consider a uniform disc of radius r and mass M. We choose the z axis to be perpendicular to the disc. Calculate the principal moments of inertia of the body about x, y and z axes respectively.

   (b) An one dimensional Simple Harmonic Oscillator is described by a Hamiltonian:

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 q^2.$$

   Consider a new set of variables (θ, I) defined as $q = \sqrt{\frac{2I}{m\omega}} \sin \theta$, and $p = \sqrt{2Im\omega} \cos \theta$.

   i. Prove that the transformation $(q, p) \to (\theta, I)$ is a canonical transformation.
   ii. Prove that I is a constant of motion and θ varies linearly with time t.
   iii. Draw the phase space trajectory of SHO in (q, p) and (θ, I) space.

6+(3+3+3)=15